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MONTE CARLO SIMULATION FOR APOLLO GO, NO-GO VERIFICATION

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MONTE CARLO SIMULATION FOR APOLLO
GO, NO-GO VERIFICATION

Bernard Kaufman

Reference 7 on p. 11

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Legends on all Graphs and Tables on pages 12 thru 21
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ABSTRACT

The purpose of this Monte Carlo study is to determine the accuracies to which the scalar orbital parameters speed, height and flight path angle can be determined from 60 seconds of ship tracking data. The Apollo Go, No-Go decision is based upon these scalar parameters. Comparisons are included with results obtained by other investigators using different methods. This study shows that speed is very sensitive to the weighting employed, and that speed and flight path angle are sensitive to tracking geometry.

CONTENTS

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I. INTRODUCTION

When the Apollo spacecraft is inserted into a parking orbit about the earth, it becomes important to determine the accuracy one might expect in computing the orbit using only a short tracking interval from a single shipboard C-band radar. The tracking results obtained by the ship are used for verification of spacecraft on board data received by telemetry for a go, no-go decision. Therefore the use of shipboard radar is that of a backup or verification system since the major decision will be based primarily on the on board position and velocity measurements (Reference 1). During the powered flight phase, coverage from ground stations is adequate but land based tracking coverage terminates at or prior to cutoff of the S-IV-B stage at about 1440 nm down-range. The insertion ship thus becomes the primary observational base for the insertion phase.

The "orbit determinations" are simulated in this report by utilizing least square fits to sampled data generated by Monte Carlo techniques. A measurement error model is first assumed and is then perturbed by a random process. The deviations are then used in a weighted least squares method to determine the "best fit" orbital elements.

Results that were obtained previously (reference 1) and results not yet published (Table 3) were at variance with one another and thus indicated that further work was necessary. This report was then undertaken as an attempt to resolve these difficulties.

II. ANALYSIS

A comment on notation should be made here. Bold face capitals denote matrices while small letters denote the elements of the matrix.

We assume a radar which measures range (r), azimuth (α) and elevation (ϵ) with corresponding errors δr, $\delta\alpha$ and $\delta\epsilon$. We may express these errors in terms of the Cartesian position vector

$$\hat{X}_{(3\times1)} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [x_i] \quad i = 1, 2, 3$$

by the first order terms of the Taylor series

$$r + \delta r = r + \sum_{i=1}^{3} \frac{\partial r}{\partial x_i} \delta x_i + v_1$$

where v_1 is a residual error term resulting from neglecting higher order terms in the Taylor's expansion.

Then

Similarly

$$\left.\begin{aligned} \delta r &= \left[\frac{\partial r}{\partial X}\right]_{(1\times 3)} \delta \hat{X}_{(3\times 1)} + v_1 \\ \delta \alpha &= \left[\frac{\partial \alpha}{\partial X}\right] \delta \hat{X} + v_2 \\ \delta \epsilon &= \left[\frac{\partial \epsilon}{\partial X}\right] \delta \hat{X} + v_3 \end{aligned}\right\} \text{ where the summation is assumed}$$

It must be pointed out here that $\delta \hat{X}$ is a time dependent matrix and therefore is not referenced to a fixed point in time. But for a least squares fit such a reference point is needed. Now

$$\delta \hat{X} = \frac{\partial \hat{X}}{\partial r} \delta r + \frac{\partial \hat{X}}{\partial \alpha} \delta \alpha + \frac{\partial \hat{X}}{\partial \epsilon} \delta \epsilon$$

where

$\delta \hat{X}$ is a 3×1 matrix.

$$\delta \hat{X}_{(3\times 1)} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

since $\dot{x}, \dot{y}, \dot{z}$ are not functions of r, α, and ϵ.

Then we may write

$$\delta \hat{X}_{(3\times 1)} = \left[\frac{\partial (x, y, z)}{\partial (r, \alpha, \epsilon)}\right]_{(3\times 3)} \begin{bmatrix} \delta r \\ \delta \alpha \\ \delta \epsilon \end{bmatrix}_{(3\times 1)} \tag{1}$$

where we consider $\delta \hat{X}$ as being expressed in an inertial coordinate system whose origin is at the center of the earth.

This equation is an inconvenient one to solve in its present form since it is time dependent and we therefore seek a variational equation in a different form which will allow us to evaluate the variation in position at any given time as a function of the measurables r, α and ϵ.

If we define a new coordinate system (z) centered at the observer where

z_1 is directed towards the East

z_2 is directed towards the North

z_3 is normal to the local tangent plane

then the coordinates of a point in space measured by the radar are given by:

$$\mathbf{Z}_{(3\times 1)} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} r \sin\alpha \cos\epsilon \\ r \cos\alpha \cos\epsilon \\ r \sin\epsilon \end{bmatrix}_{(3\times 1)}$$

For the first order terms, we have

$$\delta\mathbf{Z}_{(3\times 1)} = \begin{bmatrix} \delta z_1 \\ \delta z_2 \\ \delta z_3 \end{bmatrix} = \begin{bmatrix} \cos\epsilon \sin\alpha & -\cos\alpha & -\sin\epsilon \sin\alpha \\ \cos\epsilon \cos\alpha & \sin\alpha & -\sin\epsilon \cos\alpha \\ \sin\epsilon & 0 & \cos\epsilon \end{bmatrix}_{(3\times 3)} \begin{bmatrix} \delta r \\ -r\cos\epsilon\,\delta\alpha \\ r\,\delta\epsilon \end{bmatrix}_{(3\times 1)} \tag{2}$$

or in abbreviated form

$$\mathbf{J}^T_{(3\times 3)}\, \delta\mathbf{Z}_{(3\times 1)} = \mathbf{D}_{(3\times 3)}\, \delta\mathbf{K}_{(3\times 1)} \tag{3}$$

where J is the orthogonal 3×3 matrix;

$$\mathbf{D}_{(3\times 3)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -r\cos\epsilon & 0 \\ 0 & 0 & r \end{bmatrix}_{(3\times 3)};$$

and

$$\delta\mathbf{K}_{(3\times 1)} = \begin{bmatrix} \delta r \\ \delta\alpha \\ \delta\epsilon \end{bmatrix}_{(3\times 1)}$$

represents noise in the measurements (Reference 2).

If we let ϕ and λ be the geodetic latitude and longitude respectively of the station then we may transform equation (3) to inertial coordinates by means of:

$$\mathbf{Z}_{(3\times 1)} = \mathbf{R}_{(3\times 3)}\, \mathbf{R}_{z\,(3\times 3)}\, (\theta_G)\, \hat{\mathbf{X}}_{(3\times 1)} - \mathbf{R}_{(3\times 3)}\, \mathbf{S}_{(3\times 1)}$$

where

$$R_{(3\times 3)} = R_{x\,(3\times 3)}\left(\frac{\pi}{2} - \phi\right) R_{z\,(3\times 3)}\left(\frac{\pi}{2} + \lambda\right) ;$$

$S_{(3\times 1)}$ is the position vector of the station in an earth centered coordinate system where s_1 is towards Greenwich, s_2 is 90° east of s_1 and s_3 is along the earth's axis of rotation; R_x and R_z are rotations about the corresponding x and z axis and θ_G is the GHA at the time of observation. Using this notation equation (3) becomes

$$D_{(3\times 3)}\,\delta K_{(3\times 1)} = J^T_{(3\times 3)}\,R_{(3\times 3)}\,R_{z\,(3\times 3)}(\theta_G)\,\delta\hat{X}_{(3\times 1)} - J^T_{(3\times 3)}\,R_{(3\times 3)}\,\delta S_{(3\times 1)}$$

where $\delta\hat{X}$ is inertial. If we include in the above equation the variation of the measurement (Reference 3) with respect to bias in the measurement $(\delta\beta_{(3\times 1)})$ we may write the equation as

$$L_{(3\times 3)}\,\delta\hat{X}_{(3\times 1)} = D_{(3\times 3)}\,\delta K_{(3\times 1)} + J^T_{(3\times 3)}\,R_{(3\times 3)}\,\delta S_{(3\times 1)} - D_{(3\times 3)}\,\delta\beta_{(3\times 1)} \tag{4}$$

where

$$L_{(3\times 3)} = J^T_{(3\times 3)}\,R_{(3\times 3)}\,R_{z\,(3\times 3)}(\theta_G)$$

Equation (4) permits the computation of $\delta\hat{X}_{(3\times 1)}$ for each point in time which is then substituted into equation (1).

Consider now the position vector $\hat{X}_{(3\times 1)}$ at time $T = t$. $\hat{X}_{(3\times 1)}$ is related to the state vector $X_{0_{(6\times 1)}}$ by a functional relationship

$$\hat{X}_{(3\times 1)} = \hat{X}_{(3\times 1)}\,(X_{0_{(3\times 1)}}, \dot{X}_{0_{(3\times 1)}})$$

and for the erros $\delta\hat{X}_{(3\times 1)}$ we have

$$\hat{X}_{(3\times 1)} + \delta\hat{X}_{(3\times 1)} = \hat{X}_{(3\times 1)}\,(X_{0_{(3\times 1)}} + \delta X_{0_{(3\times 1)}}, \dot{X}_{0_{(3\times 1)}} + \delta\dot{X}_{0_{(3\times 1)}})$$

where we shall consider $X_{0_{(6\times 1)}}$ to be the state at some fixed "reference point" in time. Expanding the above as a Taylor series we have

$$\hat{X}_{(3\times 1)} + \delta\hat{X}_{(3\times 1)} = \hat{X}_{(3\times 1)}\,(X_0, \dot{X}_0) + \left[\frac{\partial\hat{X}}{\partial X_0}\right]_{(3\times 3)} \delta X_{0_{(3\times 1)}} + \left[\frac{\partial\hat{X}}{\partial\dot{X}_0}\right]_{(3\times 3)} \delta\dot{X}_{0_{(3\times 1)}} + v'_{(3\times 1)}$$

or

$$\delta \hat{X}_{(3\times 1)} = \left[\frac{\partial (x, y, z)}{\partial (x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)} \right]_{(3\times 6)} \delta X_{0_{(6\times 1)}} + V'_{(3\times 1)} \; ^{*}$$

where we do not include the partials of $\dot{x}$, $\dot{y}$, $\dot{z}$ since there is no direct measurement of them and thus $\delta \hat{X}_{(3\times 1)}$ is the matrix of position only and $\delta X_{0_{(6\times 1)}}$ is the variation in the entire state at the reference point.

Letting $\hat{\Phi}_{0_{(3\times 6)}}$ denote the matrix of partial derivatives we have

$$V'_{(3\times 1)} = \delta \hat{X}_{(3\times 1)} - \hat{\Phi}_{0_{(3\times 6)}} \delta X_{0_{(6\times 1)}}$$

or

$$V_{(3\times 1)} = L_{(3\times 3)} \delta \hat{X}_{(3\times 1)} - L_{(3\times 3)} \hat{\Phi}_{0_{(3\times 6)}} \delta X_{0_{(6\times 1)}} \tag{5}$$

where $L_{(3\times 3)}$ is defined as above and $V_{(3\times 1)} = L_{(3\times 3)} V'_{(3\times 1)}$ **. We now use the method of weighted least squares with equation (5) where we seek the best correction to the state represented by the matrix $\delta X_{0_{(6\times 1)}}$ and where $L_{(3\times 3)} \delta \hat{X}_{(3\times 1)}$ is given by equation (4). The method of least squares for the solution of the unknown parameters $\delta X_{0_{(6\times 1)}}$ stipulates that: $\Phi = V^T_{(1\times 3)} W^{-1}_{(3\times 3)} V_{(3\times 1)}$ be a minimum

where
$$W = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & r^2 \cos^2 \epsilon \, \sigma_\alpha^2 & 0 \\ 0 & 0 & r^2 \sigma_\epsilon^2 \end{bmatrix}_{(3\times 3)} \tag{6}$$

is the weighting matrix.

---

*This matrix of partial derivatives is called the state transition matrix when the complete matrix is written as

$$\left[\frac{\partial (x, y, z, \dot{x}, \dot{y}, \dot{z})}{\partial (x_0, y_0, z_0, \dot{x}, \dot{y}_0, \dot{z}_0)} \right]_{(6\times 6)}$$

The computation of this matrix is found in reference 4.

**It should be noted that equation (4) may now be written as:

$$L_{(3\times 3)} \hat{\Phi}_{0_{(3\times 6)}} \delta X_{0_{(6\times 1)}} = D_{(3\times 3)} \delta K_{(3\times 1)} + J^T_{(3\times 3)} R_{(3\times 3)} \delta S_{(3\times 1)} - D_{(3\times 3)} \delta \beta_{(3\times 1)}$$

where the fixed reference point $(\delta X_{0_{(6\times 1)}})$ is necessary for the least squares procedures.

Substituting from (4)

$$\Phi = [L\delta\hat{X} - L\hat{\Phi}_0\,\delta X_0]^T W^{-1}\,[L\delta\hat{X} - L\hat{\Phi}_0\,\delta X_0]$$

$$= [\delta\hat{X}^T L^T - \delta X_0^T\,\hat{\Phi}_0^T L^T]\,W^{-1}\,[L\delta\hat{X} - L\hat{\Phi}_0\,\delta X_0]$$

$$= \delta\hat{X}^T L^T W^{-1} L\delta\hat{X} - \delta X_0^T\,\hat{\Phi}_0^T L^T W^{-1} L\delta\hat{X} - \delta\hat{X}^T L^T W^{-1} L\hat{\Phi}_0\,\delta X_0 + \delta X_0^T\,\hat{\Phi}_0^T L^T W^{-1} L\hat{\Phi}_0\,\delta X_0$$

then

$$\frac{\partial\Phi}{\partial(\delta X_0)} = 0 = -\hat{\Phi}_0^T L^T W^{-1} L\delta\hat{X} + \hat{\Phi}_0^T L^T W^{-1} L\hat{\Phi}_0\,\delta X_0$$

or

$$\hat{\Phi}_0^T L^T W^{-1} L\delta\hat{X} = \hat{\Phi}_0^T L^T W^{-1} L\hat{\Phi}\,\delta X_0$$

If we write the above in summation notation where we sum over n observations we have

$$\sum_1^n(\hat{\Phi}_0^T L^T W^{-1} L\delta\hat{X})_{(6\times 1)} = \left[\sum_1^n(\hat{\Phi}_0^T L^T W^{-1} L\hat{\Phi}_0)_{(6\times 6)}\right]\delta X_{0_{(6\times 1)}}$$

or

$$\delta X_{0_{(6\times 1)}} = \left[\sum_1^n(\hat{\Phi}_0^T L^T W^{-1} L\hat{\Phi}_0)\right]^{-1}_{(6\times 6)}\left[\sum_1^n(\hat{\Phi}_0^T L^T W^{-1} L\delta\hat{X})\right]_{(6\times 1)} \tag{7}$$

which are the normalized equations.

Equation 7 represents the weighted least squares procedure for determining the "best fit" variations at a fixed reference point. The nonweighted procedure is identical and if one sets $W = I$, the identity matrix, equation (7) reduces to the nonweighted case:

$$\delta X_{0_{(6\times 1)}} = \left[\sum_1^n(\hat{\Phi}_0^T\,\hat{\Phi}_0)\right]^{-1}_{(6\times 6)}\left[\sum_1^n(\hat{\Phi}_0^T\,\delta\hat{X})\right]_{(6\times 1)} \tag{8}$$

III. TRANSFORMATION TO SPHERICAL ELEMENTS

Once $\delta X_{0_{(6\times 1)}}$ is obtained either by equation (7) or equation (8) it may be more instructive to look at the resulting uncertainties in terms of deviations in the insertion elements of the spacecraft. Accordingly, we define these elements as:

h_i spacecraft altitude above the earth

v_i speed

γ_i flight path angle

α_i insertion azimuth

λ_i right ascension (or longitude)

δ_i declination (or latitude)

The transformation from injection elements to position and velocity coordinates are as follows (References 2 and 5):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{(3\times 1)} = \begin{bmatrix} r \cos \delta_i \cos \lambda_i \\ r \cos \delta_i \sin \lambda_i \\ r \sin \delta_i \end{bmatrix}_{(3\times 1)}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{(3\times 1)} \begin{bmatrix} v_i (\sin\gamma_i \cos\delta_i \cos\lambda_i - \cos\gamma_i \sin\alpha_i \sin\lambda_i - \cos\gamma_i \cos\alpha_i \sin\delta_i \cos\lambda_i) \\ v_i (\sin\gamma_i \cos\delta_i \sin\lambda_i + \cos\gamma_i \sin\alpha_i \cos\lambda_i - \cos\gamma_i \cos\alpha_i \sin\delta_i \sin\lambda_i) \\ v_i (\sin\gamma_i \sin\delta_i + \cos\gamma_i \cos\alpha_i \cos\delta_i) \end{bmatrix}_{(3\times 1)} \tag{9}$$

It can easily be seen that we may then write the transformation for the deviations as

$$\delta X_{0_{(6\times 1)}} = \left[\frac{\partial (x, y, z, \dot{x}, \dot{y}, \dot{z})}{\partial (h_i, v_i, \gamma_i, \alpha_i, \lambda_i, \delta_i)} \right]_{(6\times 6)} \begin{bmatrix} \delta h_i \\ \delta v_i \\ \delta \gamma_i \\ \delta \alpha_i \\ \delta \lambda_i \\ \delta (\delta_i) \end{bmatrix}_{(6\times 1)}$$

or

$$\delta X_{0_{(6\times 1)}} = \Psi_{(6\times 6)} \beta_{(6\times 1)} \tag{10}$$

where $\Psi_{(6\times 6)}$ is the matrix of partials and $\beta_{(6\times 1)}$ is the variational matrix. Then we have

$$\beta_{(6\times 1)} = \Psi^{-1}_{(6\times 6)} \delta X_{0_{(6\times 6)}}$$

and from equation (7)

$$\beta_{(6\times 1)} = \Psi^{-1}_{(6\times 6)} \left[\sum_1^n (\hat{\Phi}_0^T L^T W^{-1} L \hat{\Phi}_0) \right]^{-1}_{(6\times 6)} \left[\sum_1^n (\hat{\Phi}_0^T L^T W^{-1} L \delta \hat{X}) \right]_{(6\times 1)} \tag{11}$$

Recall from equation 4 that

$$L_{(3\times 3)} \delta \hat{X}_{(3\times 1)} = D_{(3\times 3)} \delta K_{(3\times 1)} + J^T_{(3\times 3)} R_{(3\times 3)} \delta S_{(3\times 1)} - D_{(3\times 3)} \delta \beta_{(3\times 1)}$$

If we apply random numbers to the error terms $\delta K_{(3\times 1)}$, $\delta S_{(3\times 1)}$ and $\delta \beta_{(3\times 1)}$ we then have a means of calculating the statistics in a Monte Carlo sense. This is done in the following way: random numbers are generated and used as multipliers for the $\delta S_{(3\times 1)}$ and $\delta \beta_{(3\times 1)}$ errors. These products then give modified $\delta S_{(3\times 1)}$ and $\delta \beta_{(3\times 1)}$ which are constants over the n observations. However for each of the n observations different random numbers are generated for the $\delta K_{(3\times 1)}$ or noise errors. Then equations (5) and (7) allow us to compute the variations. This procedure is repeated for j times or Monte Carlo samples yielding j matrices for δX_0 and β. The standard deviations may then be computed. For example

$$\sigma_{h_i} = \left[\frac{\sum^{j} (\delta h_i)^2}{j - 1} - (\delta \overline{h_i})^2 \right]^{1/2}$$

where $\overline{\delta h_i}$ is the mean of the errors in altitude determined from j Monte Carlo runs. Similarly for the other injection parameters and for the uncertainties in the state δX_0.

IV. CALCULATION OF PERIGEE UNCERTAINTIES

Deviations in perigee may be included in the Monte Carlo statistics by means of classical methods of celestial mechanics. Denote the original "unperturbed" reference state by X_0. Then the "best fit" state is

$$X_{(6\times 1)} = X_{0_{(6\times 1)}} + \delta X_{0_{(6\times 1)}} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad \text{and} \quad \begin{aligned} r &= (x^2 + y^2 + z^2)^{1/2} \\ v &= (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \end{aligned}$$

Then perigee is calculated as follows

$$a = \frac{r}{2 - \dfrac{r v^2}{\mu}} \tag{12}$$

where μ is the gravitational constant of the earth

$$\left. \begin{aligned} e \sin E &= \frac{\vec{r} \cdot \vec{v}}{\sqrt{\mu a}} \\ e \cos E &= 1 - r/a \end{aligned} \right\} \tag{13}$$

which yield the eccentricity e and finally

$$r_p = a(1 - e) \tag{14}$$

If perigee is also computed with the original vector X_0 then the variation of perigee is thus obtained and the standard deviation may be calculated in the usual manner. It must be pointed out that for near circular orbits, perigee uncertainties are not gaussian. Thus the standard deviation in this case is not really what it implies. However, once one realizes this, the standard deviation and the mean for the perigee errors still allow one to intuitively gain insight into what is happening.

V. EXAMPLES AND RESULTS

Two different sets of data were selected for numerical examples in a preliminary study of the effects of geometry on the uncertainties. Range safety requirements at Cape Kennedy enforce certain restrictions on launching and to meet these requirements two bands of 26° launch azimuth widths were selected. The azimuth spread of 26° represents the maximum daily launch window for the Apollo mission.

Two ship locations are necessary to provide coverage of the two azimuth bands. Ship A with geodetic coordinates 26° .0 (latitude North) and 47° .5 (longitude West) is used to cover the azimuth band from 72° to 98°. Ship B with coordinates 21° .25 N and 48° .75 W is used to cover the azimuth band from 82° to 108°.

Figure 1 shows the locations of the ships and the two bands of azimuth. The coverage of the radar indicated by the circles are for elevation angles above 5°.

Table 1 shows the uncertainties for the weighted least squares for height, speed, flight path angle and perigee. The last three columns are the average perigee uncertainties and the uncertainties in total position and velocity. The first 3 columns are launch azimuth, elevation angle of the first observation and elevation angle of the last observation. As can easily be seen, the uncertainties for Ship A and Ship B are almost identical. The time span for the observations is exactly one minute along the orbit with the first observation occurring at insertion into the orbit and one observation per second thereafter.

Table 2 is the same as Table 1 except that here the weighting matrix is the identity matrix. This has the effect of giving a heavier weight to the angular measurements than is given the more accurate range measurement and one would suspect that the resulting uncertainties would be larger. A comparison of Tables 1 and 2 shows that the uncertainties in h_i, γ_i and total position are the same; however, for speed, the difference is considerable. The deviations associated with perigee and total velocity are also larger in the unweighted case. This clearly shows the penalty one pays for using nonweighted least squares.

Table 3 is the results obtained by W. D. Kahn in a study (yet to be published—Ref. 8) using a linear error analysis program. A comparison with Table 1 for ship B shows that speed, flight path angle and the total velocity are in fairly good agreement. However altitude and total position do differ by a significant amount.

P. G. Brumberg in reference 1 shows results that he obtained using Monte Carlo techniques. A complete description of the error model used can be found in the reference. A comparison between the results obtained by Kahn, Brumberg and the author is shown in Table 4 for the 108 degree launch azimuth only. The results obtained for Brumberg were read from graphs in reference 1. These results were obtained without weighting the measurements.

The above mentioned results show that while the author and Mr. Kahn's results agree except in altitude and position further investigation is still required into this difference.

As pointed out earlier, the uncertainties in perigee for a near circular orbit are not distributed in a gaussian manner. However in the above mentioned examples, the uncertainties in the state vector were normally distributed with a zero mean within the expected accuracy of a finite sample size.

Several other studies have been made in connection with the perigee uncertainties. Reference 6 utilizes an analytical expression of the go, no-go criterion based on lifetime and results are given in that report. A preliminary analysis, currently being prepared for publication (Reference 7), has also been undertaken for the probability distribution of perigee uncertainties and its application to the go, no-go decision. This analysis utilizes two-body equations of motion.

Figures 2 through 7 show the uncertainties in h_i, v_i, γ_i, r_p, total position and total velocity for Ships A and B plotted versus launch azimuth. Both weighted and unweighted cases were shown. The effect of geometry is clearly seen in these graphs.

The error model used as input in the above examples was as follows:

	Noise (δK)	Bias ($\delta \beta$)
Range (meters)	10.0	20.0
Azimuth (milliradians)	0.4	0.8
Elevation (milliradians)	0.4	0.8

STATION ERRORS (meters)

X	430
Y	430
Z	0

WEIGHTING MODEL FOR WEIGHTED CASES

σ range = 10.0 meters

σ azimuth = 0.4 milliradians

σ elevation = 0.4 milliradians

VI. ACKNOWLEDGEMENTS

The author wishes to acknowledge many helpful discussions with Mr. R. T. Groves and Mr. W. D. Kahn which pointed out many pitfalls in the methods used for this study. Many thanks are also due to Mr. C. R. Newman for his help in the programming of this problem.

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8. Private conversations with Mr. Kahn.

Table 1
Standard Deviations Weighted Least Squares

One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 43°.75 (Long. West)

Launch Azimuth	Initial Elevation (deg)	Final Elevation (deg)	σ_h (km)	σ_v (m/sec)	σ_γ (deg)	$\sigma_{\Delta r_p}$ (km)	Average Δr_p (km)	Total Position (km)	Total Velocity (m/sec)
Ship A									
72	7.5	9.1	.93	.612	.022	2.3	-2.5	1.28	6.04
75	8.4	12.8	.90	.513	.026	2.4	-2.6	1.23	6.39
80	11.9	23.7	.77	.364	.030	2.4	-2.8	1.06	6.80
85	14.1	43.4	.74	.284	.042	2.9	-4.0	1.00	7.99
90	13.1	30.7	.74	.322	.032	2.5	-3.0	1.00	6.80
95	9.8	15.9	.85	.445	.026	2.4	-2.5	1.13	6.27
98	7.7	11.1	.94	.550	.025	2.5	-2.6	1.23	6.19
Ship B									
82	7.6	10.1	.94	.583	.024	2.3	-2.5	1.27	6.13
85	8.6	13.9	.90	.492	.028	2.4	-2.7	1.22	6.50
90	11.8	25.7	.79	.365	.033	2.5	-3.1	1.07	7.06
95	13.2	40.2	.77	.297	.043	3.1	-4.0	1.03	8.02
97	13.0	36.2	.76	.307	.038	2.9	-3.6	1.03	7.47
100	11.7	25.4	.79	.357	.032	2.6	-2.9	1.06	6.79
105	8.5	13.7	.91	.495	.027	2.5	-2.6	1.19	6.35
108	7.5	9.8	.96	.590	.023	2.4	-2.5	1.25	6.04

	Bias ($\sigma\beta$)	Noise (σK)	Station errors (meters)
r (meters)	20.0	10.0	x 430.0
α (milliradians)	0.8	0.4	y 430.0
ϵ (milliradians)	0.8	0.4	z 0.0

Table 2
Standard Deviations Un-Weighted Least Squares

One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)

Launch Azimuth	Initial Elevation (deg)	Final Elevation (deg)	σ_h (km)	σ_v (m/sec)	σ_γ (deg)	σ_{r_p} (km)	Average Δr_p (km)	Total Position (km)	Total Velocity (m/sec)
Ship A									
72	7.5	9.1	.94	2.83	.022	5.3	-4.4	1.29	6.68
75	8.4	12.8	.90	2.37	.027	4.4	-4.1	1.24	7.02
80	11.8	23.7	.79	1.68	.034	3.3	-3.8	1.08	7.54
85	14.1	43.4	.74	.770	.045	3.3	-4.4	1.01	8.34
90	13.1	30.7	.77	1.39	.038	3.2	-4.1	1.03	7.58
95	9.8	15.9	.86	2.19	.028	3.6	-4.2	1.14	6.84
98	7.7	11.1	.95	2.60	.026	4.2	-4.7	1.24	6.79
Ship B									
82	7.6	10.1	.94	2.70	.024	5.1	-4.3	1.28	6.76
85	8.6	13.9	.90	2.23	.029	4.1	-4.0	1.23	7.15
90	11.8	25.7	.80	1.48	.038	3.2	-4.0	1.09	7.80
95	13.2	40.2	.77	.731	.046	3.4	-4.5	1.04	8.35
97	13.0	36.2	.78	.890	.044	3.3	-4.3	1.04	8.10
100	11.7	25.4	.81	1.50	.037	3.3	-4.1	1.07	7.50
105	8.5	13.7	.92	2.29	.029	3.8	-4.4	1.20	6.93
108	7.5	9.8	.96	2.78	.023	4.5	-4.8	1.25	6.65

	Bias ($\delta\beta$)	Noise (δK)	Station errors (meters)	
r (meters)	20.0	10.0	x	430.0
α (milliradians)	0.8	0.4	y	430.0
ϵ (milliradians)	0.8	0.4	z	0.0

Table 3
Linear Error Analysis (Kahn)

One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from
a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)

Launch Azimuth	σ_h (km)	σ_v (m/sec)	σ_γ (deg)	Total Position (km)	Total Velocity (m/s)
Ship B					
82	.63	.66	.024	1.11	6.85
85	.50	.56	.025	.95	6.91
90	.30	.41	.030	.76	7.33
95	.18	.41	.040	.68	8.25
100	.30	.42	.030	.76	7.33
108	.63	.67	.024	1.11	6.87

	Bias ($\sigma\beta$)	Noise (σK)	Station errors (meters)	
r (meters)	20.0	10.0	x	430.0
α (milliradians)	0.8	0.4	y	430.0
ϵ (milliradians)	0.8	0.4	z	0.0

Table 4
108° Launch Azimuth

One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from
a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)

	σ_v (m/sec)	σ_γ (deg)	σ_h (km)	Total Position (km)	Total Velocity (m/sec)
Kahn	.67	.024	.63	1.11	6.87
Brumberg	2.1	.03	.63	1.5	8.3
Kaufman (weighted)	.59	.023	.96	1.25	6.04
Kaufman (unweighted)	2.78	.023	.96	1.25	6.65

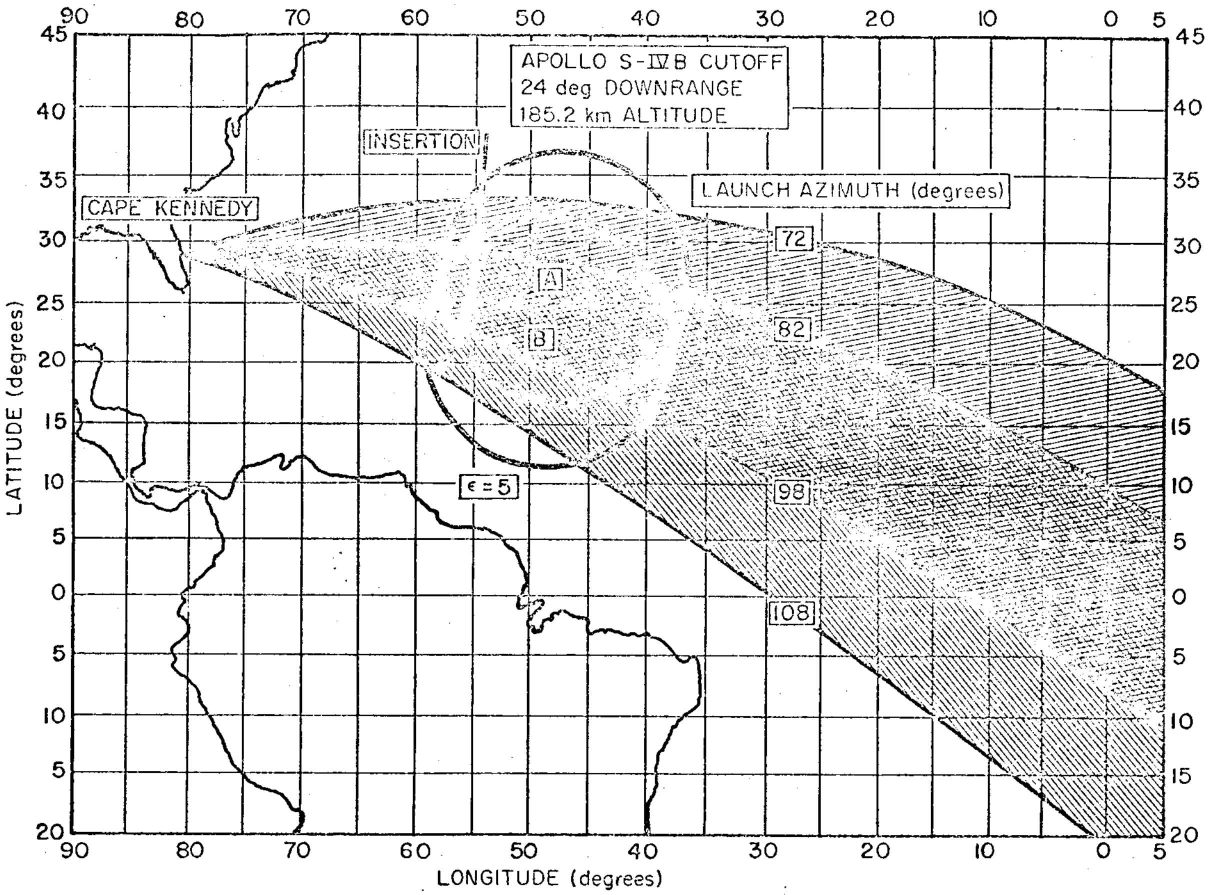
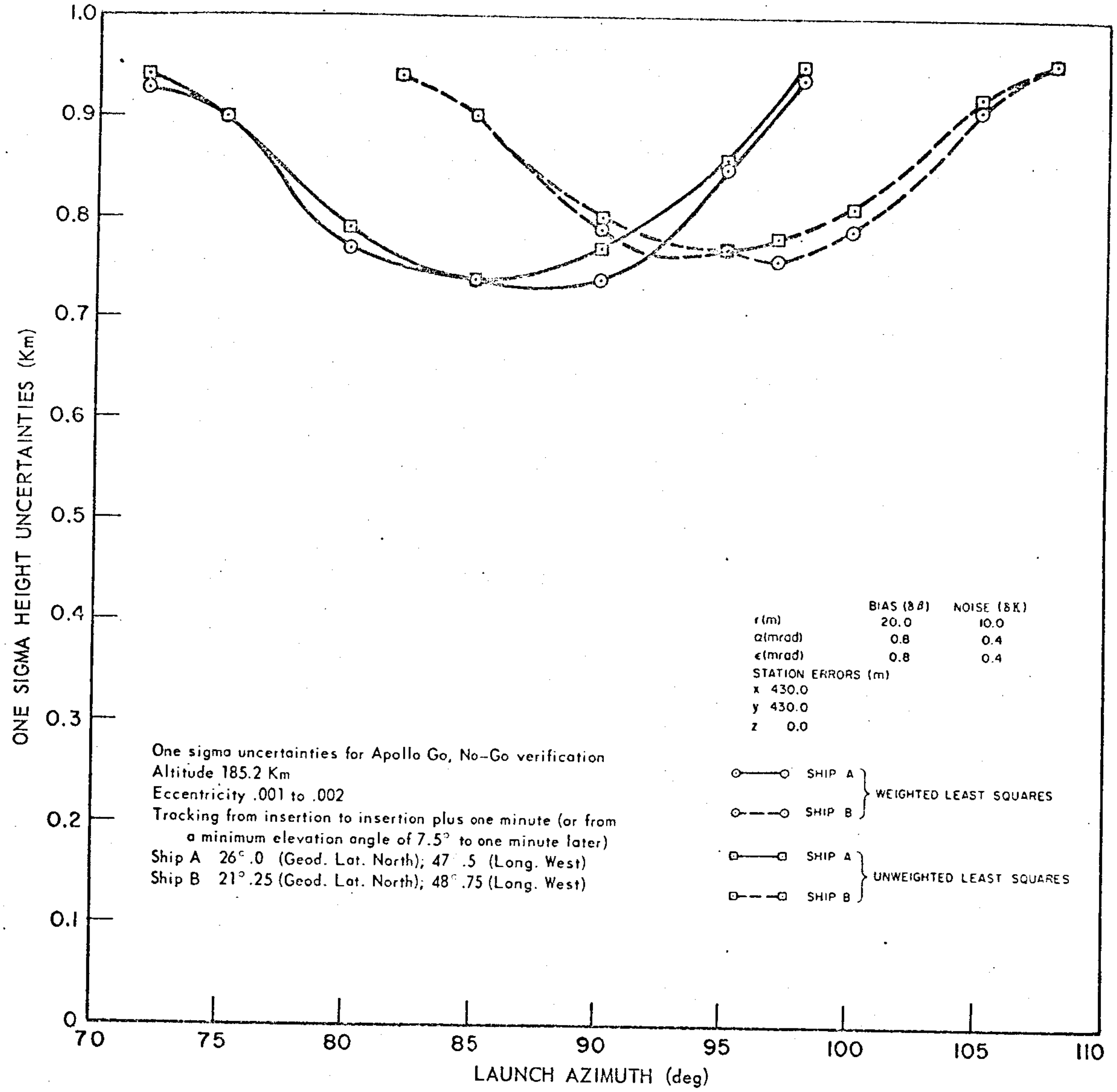

APOLLO S-IVB CUTOFF
24 deg DOWNRANGE
185.2 km ALTITUDE
INSERTION
CAPE KENNEDY
LAUNCH AZIMUTH (degrees)
72
A
B
82
98
108
ε = 5
LATITUDE (degrees)
LONGITUDE (degrees)

Figure 1



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
ONE SIGMA HEIGHT UNCERTAINTIES (Km)
70
75
80
85
90
95
100
105
110
LAUNCH AZIMUTH (deg)
BIAS (δβ) NOISE (δK)
r(m) 20.0 10.0
a(mrad) 0.8 0.4
ε(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B
UNWEIGHTED LEAST SQUARES

Figure 2

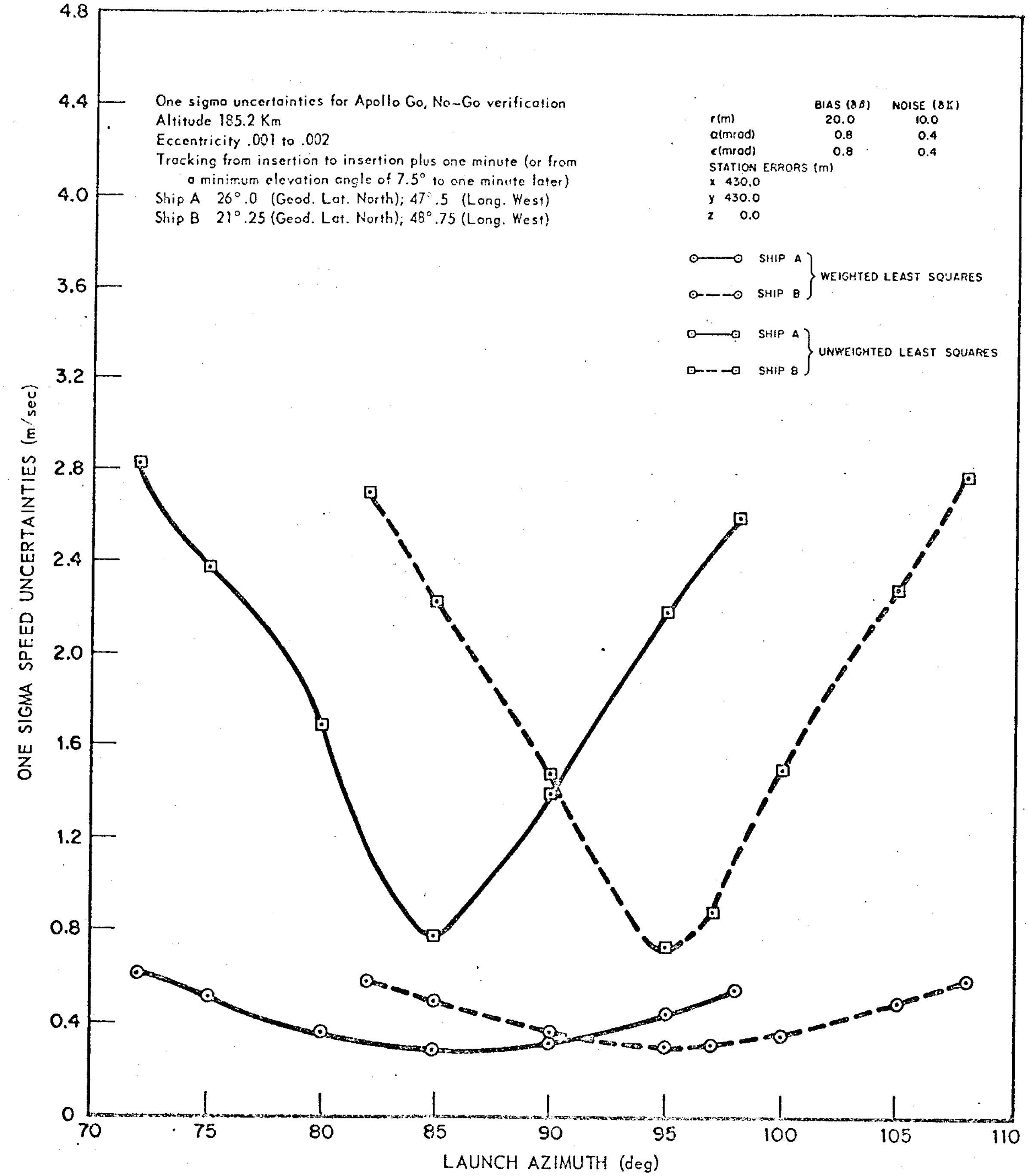

One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)
BIAS (δβ) NOISE (δK)
r(m) 20.0 10.0
α(mrad) 0.8 0.4
ε(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B
UNWEIGHTED LEAST SQUARES
ONE SIGMA SPEED UNCERTAINTIES (m/sec)
LAUNCH AZIMUTH (deg)

Figure 3

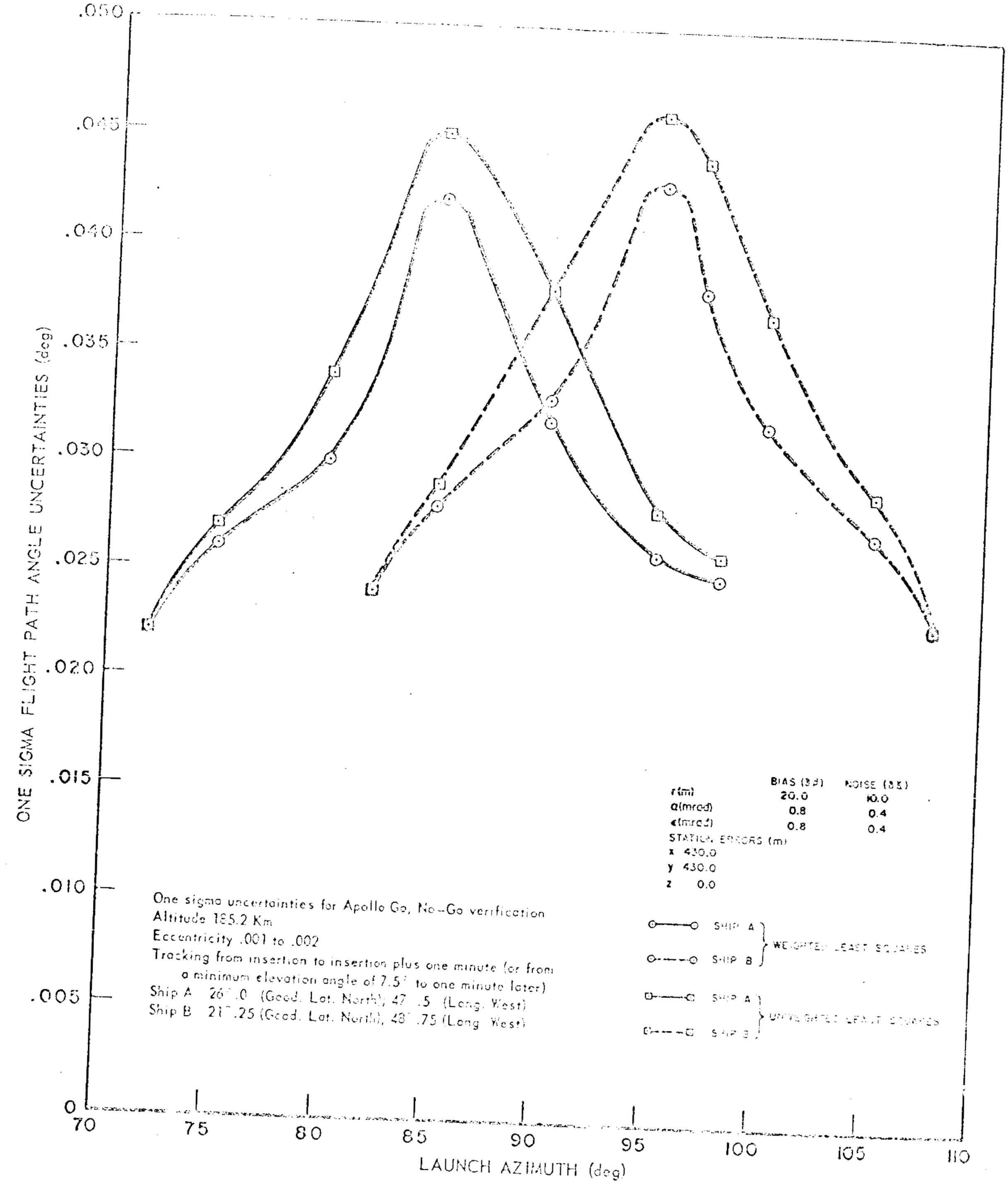
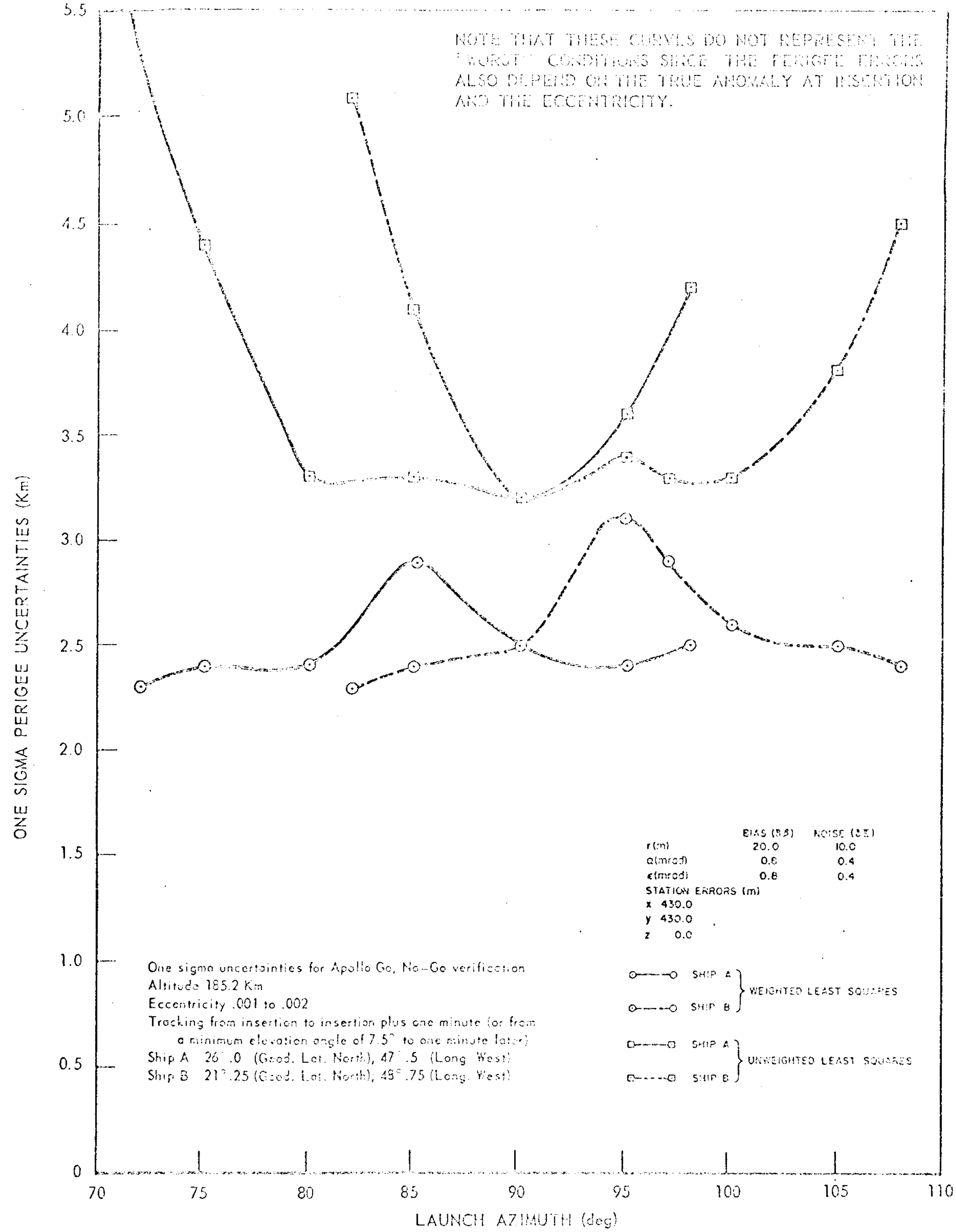

ONE SIGMA FLIGHT PATH ANGLE UNCERTAINTIES (deg)
.050
.045
.040
.035
.030
.025
.020
.015
.010
.005
0
70
75
80
85
90
95
100
105
110
LAUNCH AZIMUTH (deg)
BIAS (3σ)
NOISE (3σ)
r(m) 20.0 10.0
α(mrad) 0.8 0.4
ε(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North), 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North), 48°.75 (Long. West)
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B

Figure 4



NOTE THAT THESE CURVES DO NOT REPRESENT THE "WORST" CONDITIONS SINCE THE PERIGEE ERRORS ALSO DEPEND ON THE TRUE ANOMALY AT INSERTION AND THE ECCENTRICITY.
ONE SIGMA PERIGEE UNCERTAINTIES (Km)
LAUNCH AZIMUTH (deg)
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
70
75
80
85
90
95
100
105
110
BIAS (3σ) NOISE (3σ)
r(m) 20.0 10.0
α(mrad) 0.8 0.4
ε(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B
UNWEIGHTED LEAST SQUARES
One sigma uncertainties for Apollo Go, No–Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North), 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North), 48°.75 (Long. West)

Figure 5

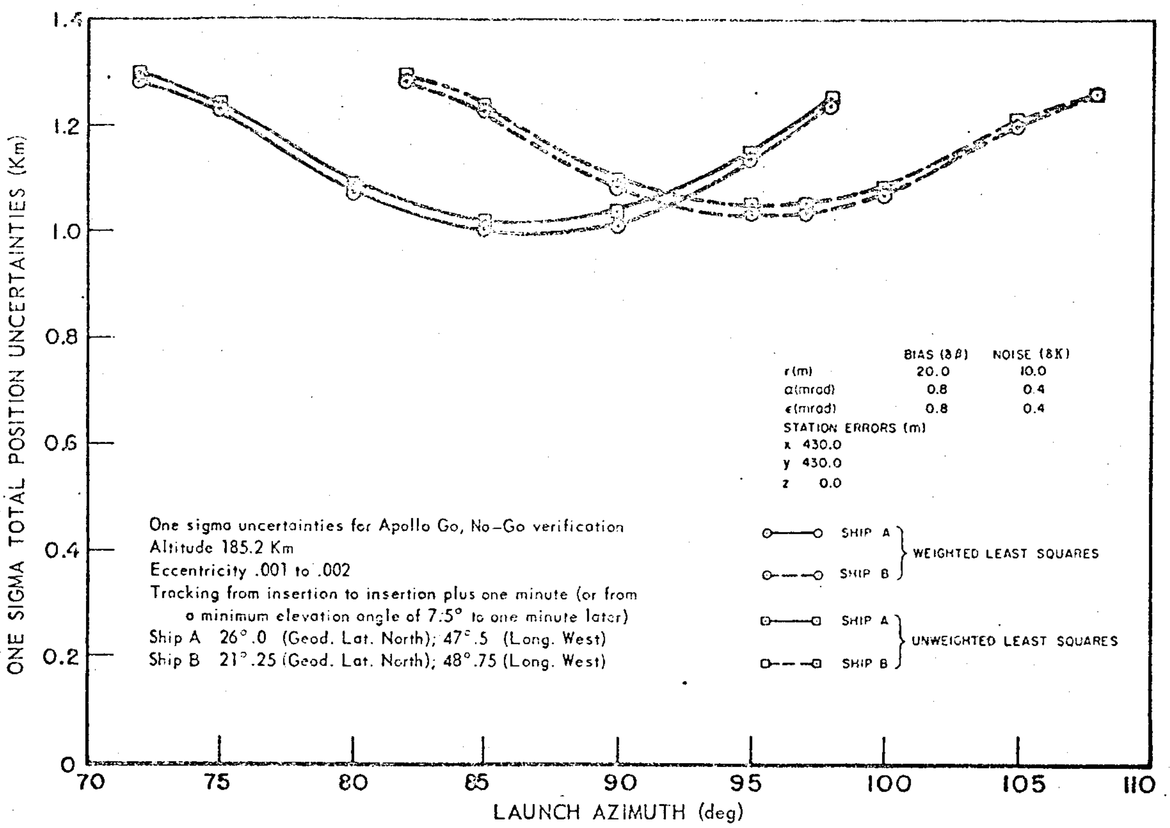

ONE SIGMA TOTAL POSITION UNCERTAINTIES (Km)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
70
75
80
85
90
95
100
105
110
LAUNCH AZIMUTH (deg)
BIAS (δβ) NOISE (δK)
r(m) 20.0 10.0
α(mrad) 0.8 0.4
ϵ(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B
UNWEIGHTED LEAST SQUARES

Figure 6

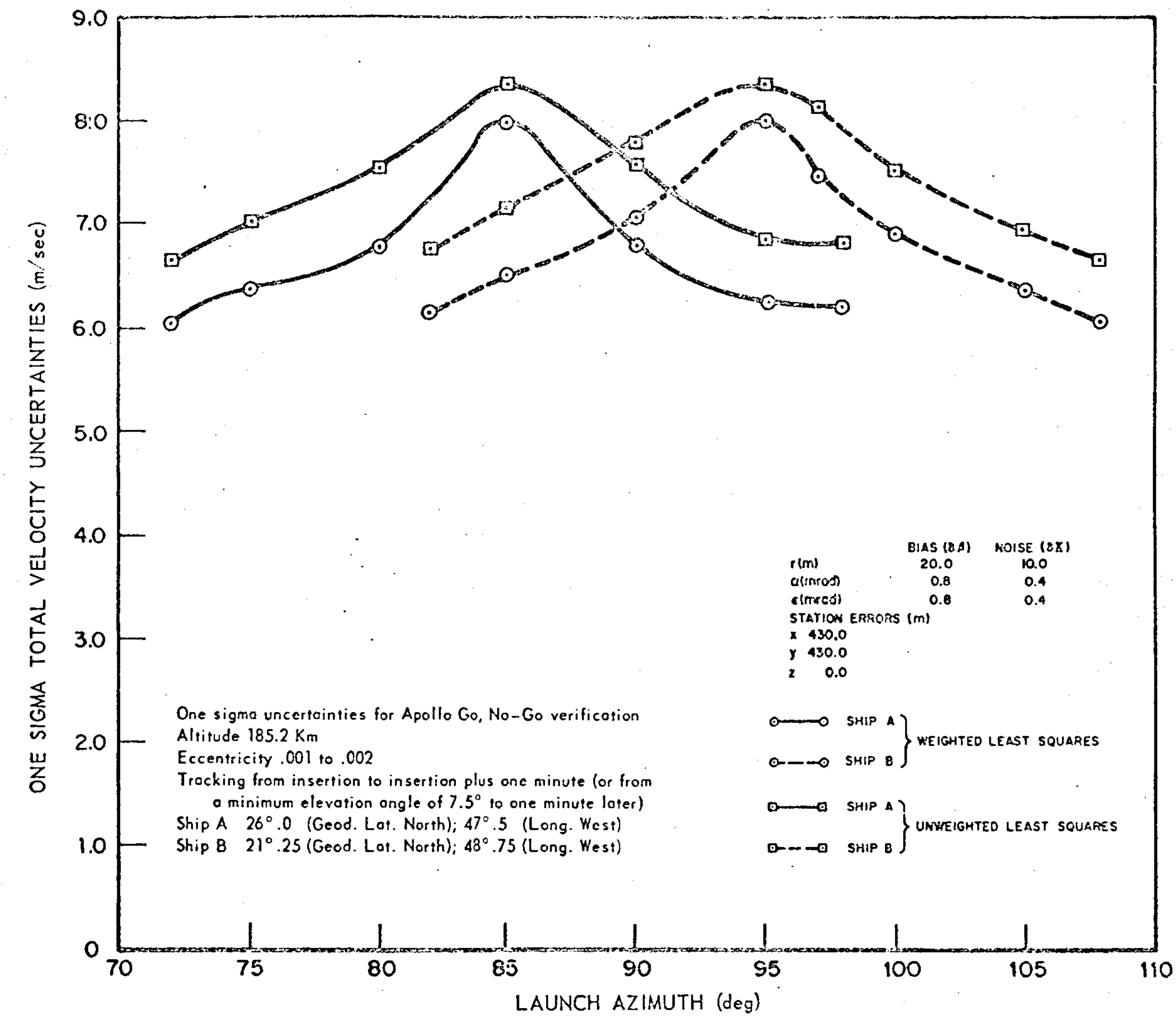

9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
ONE SIGMA TOTAL VELOCITY UNCERTAINTIES (m/sec)
70
75
80
85
90
95
100
105
110
LAUNCH AZIMUTH (deg)
BIAS (δβ) NOISE (δX)
r(m) 20.0 10.0
α(mrad) 0.8 0.4
ε(mrad) 0.8 0.4
STATION ERRORS (m)
x 430.0
y 430.0
z 0.0
One sigma uncertainties for Apollo Go, No-Go verification
Altitude 185.2 Km
Eccentricity .001 to .002
Tracking from insertion to insertion plus one minute (or from a minimum elevation angle of 7.5° to one minute later)
Ship A 26°.0 (Geod. Lat. North); 47°.5 (Long. West)
Ship B 21°.25 (Geod. Lat. North); 48°.75 (Long. West)
SHIP A
SHIP B
WEIGHTED LEAST SQUARES
SHIP A
SHIP B
UNWEIGHTED LEAST SQUARES

Figure 7